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Sum rule for e$^+$-e$^-$ annihilation and U(3) ⊗ U(3) symmetry breaking

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Abstract. It is argued that the difference between the production cross section for $T = 0$, and $T = 1$ hadronic final states is related to the U(3) ⊗U(3) symmetry breaking. A simple sum rule is derived. Various symmetry breaking models are discussed. It is found that results from saturation of sum rule with vector mesons favour our previously obtained values $b = c \simeq -1$ for the U(3) ⊗U(3) symmetry breaking parameters.

1. Introduction

It has been recognized quite early that the e$^+$–e$^-$ annihilation cross section offers a unique opportunity to measure the behaviour of the spectrum function of the electromagnetic current (Dooher 1967, Bjorken 1966, Gribov *et al.* 1967, Sakurai 1969, Gatto 1969 and Pais and Treiman 1970). Various behaviours have been associated with the energy dependence of the total e$^+$–e$^-$ annihilation cross section:

$$\sigma(s) = \frac{16\pi^3\alpha^2}{s^2}(\rho^{33}(s)+\tfrac{1}{3}\rho^{88}(s)) \tag{1.1}$$

due to different predictions for the spectrum function ρ^{33}, ρ^{88} under different models. Hence the $\rho^{33}(s)$ or $\rho^{88}(s)$ by themselves, as derived from e$^+$–e$^-$ annihilation, are capable of differentiating the current algebra from the field algebra. It is our intention to show that the difference $\rho^{33}(s)-\rho^{88}(s)$ of the spectrum functions offers us a chance to determine the U(3)⊗U(3) symmetry breaking parameters, if the algebra for the SU(3) × SU(3) symmetry is agreed upon.

Specifically let us confine ourselves to the symmetry breaking model which is proposed by Gell-Mann (1964) and Gell-Mann *et al.* (1968).

$$\mathscr{H} = \mathscr{H}_0 - S_0 - cS_8 \tag{1.2}$$

where $\mathscr{H}_0$ is the U(3)⊗U(3) invariant term, and S_0, S_8 are the singlet and the eighth component of the scalar density. In § 2 we are able to derive a sum rule for the e$^+$–e$^-$ cross sections within the context of equation (1.2):

$$\frac{1}{16\pi^3\alpha^2}\int_0^\infty \mathrm{d}s\, s^2(\sigma_{T=1}(s)-3\sigma_{T=0}(s)) = \bar{\alpha}F_\pi{}^2 m_\pi{}^2 + \frac{2}{\sqrt{3}}\langle Z_8\rangle \tag{1.3}$$

where

$$\bar{\alpha} = \frac{1}{\sqrt{2}}\frac{b+c-bc/\sqrt{2}}{(1+c/\sqrt{2})(1+b/\sqrt{2})}$$

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and

$$b = \frac{\langle S_8 \rangle}{\langle S_0 \rangle}.$$

The quantity $\langle Z_8 \rangle$ is found to be rather model dependent and is evaluated under three different considerations in § 3: (i) asymptotic SU(3) symmetry; (ii) vector meson saturation model and (iii) free quark model. In § 4 the value $\bar{\alpha}$ is calculated for different estimates of bc from different authors (Gell-Mann *et al.* 1968, Okubo and Mathur 1970a, 1970b, Mathur *et al.* 1970, Kamal 1970 and Lai and Lo 1970, 1971). They range widely from $\bar{\alpha} = 0{\cdot}0$ to approximately -20, and so presumably it might be feasible to differentiate which is the correct one experimentally. If the vector mesons are assumed to dominate the spectrum functions, one can get an independent estimate of $\bar{\alpha}$. Our calculation shows that it supports the estimate of Lai and Lo (1970, 1971) for $b = c \simeq -1$.

2. e^+–e^- cross section, symmetry breaking and sum rule

In this section we shall first relate the $e^+ + e^-$ total hadronic cross section to the spectrum function. Secondly the spectrum function is expressed in terms of commutation relations using the technique of Bjorken's limit. Thirdly the commutation relation is evaluated in the symmetry breaking model. Combining these three together we get a sum rule for the e^+–e^- cross section, which can be subjected to experimental tests.

The e^+–e^- cross section for hadron production is given, to the lowest order of electromagnetic interaction, by

$$\sigma = \frac{1}{2} \sum_{n,i} (2\pi)^3 \delta^4(p_1 + p_2 - p_n) |M_{ni}|^2 \tag{2.1}$$

where

$$M_{ni} = \frac{e^4}{q^4} \langle 0 | j_\mu^{e} | e^+ e^- \rangle \langle e^+ e^- | j_\nu^{e} | 0 \rangle \langle n | j_\mu^{\gamma}(0) | 0 \rangle \langle 0 | j_\nu^{\gamma}(0) | n \rangle. \tag{2.2}$$

The notation we have used is: j_μ^{e} is the electron current, j_μ^{γ} is the hadronic current for electromagnetic interaction

$$j_\mu^{\gamma} = j_\mu^{3} + \frac{1}{\sqrt{3}} j_\mu^{8}. \tag{2.3}$$

p_1, p_2 are the four momenta of the initial electron and position, p_n is the four momentum of the final hadron state $|n\rangle$, and $q = p_1$ and p_2. The cross section becomes

$$\sigma = \frac{\pi e^4}{q^4} L_{\mu\nu} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \rho^{\gamma}(-q^2) \tag{2.4}$$

where

$$L_{\mu\nu} = \tfrac{1}{4} \sum_{\text{spin}} \langle 0 | j_\mu^{e} | e^+ e^- \rangle \langle e^+ e^- | j_\nu^{e} | 0 \rangle \tag{2.5}$$

$$\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \rho^{\alpha\beta}(-q^2) = \sum (2\pi)^3 \delta^4(q - p_n) \langle 0 | j_\nu^{\alpha}(0) | n \rangle \langle n | j_\mu^{\beta}(0) | 0 \rangle \tag{2.6}$$

for $\alpha, \beta = 0, 3, 8$, and

$$\rho^{\gamma}(-q^2) = \rho^{33}(-q^2) + \tfrac{1}{3} \rho^{88}(-q^2). \tag{2.6a}$$

The tensor $L_{\mu\nu}$ has the following properties:

$$q_\mu L_{\mu\nu} = 0 \tag{2.7}$$

and

$$L_{\mu\mu} = 1 \qquad m_e \simeq 0. \tag{2.7a}$$

We finally arrive at the formulae for the cross sections

$$\sigma = \frac{16\pi^3\alpha^2}{q^4}\,\rho^\gamma(-q^2) \tag{2.8}$$

$$\sigma_{T=1} = \frac{16\pi^3\alpha^2}{q^4}\,\rho^{33}(-q^2) \tag{2.8a}$$

$$\sigma_{T=0} = \frac{16\pi^3\alpha^2}{3q^4}\,\rho^{88}(-q^2). \tag{2.8b}$$

Let us now turn our attention to the consideration of the vacuum expectation value of the time ordered product of two vector currents:

$$\begin{aligned}\Delta_{\mu\nu}^{\alpha\beta} &= \mathrm{i}\int \mathrm{d}^4x \exp(-\mathrm{i}q.x)\langle 0|T(j_\mu{}^\alpha(x)j_\nu{}^\beta(0))|0\rangle \\ &= \int_0^\infty \frac{\mathrm{d}m^2}{q^2+m^2-\mathrm{i}\epsilon}\left\{\left(\delta_{\mu\nu}+\frac{q_\mu q_\nu}{m^2}\right)\rho^{\alpha\beta}(m^2)\right\} - \delta_{\mu 4}\delta_{\nu 4}\int_0^\infty \frac{\mathrm{d}m^2}{m^2}\rho^{\alpha\beta}(m^2)\end{aligned} \tag{2.9}$$

for α, $\beta = 0, 3, 8$. Taking the Bjorken's limit $q_0 \to \infty$, we get

$$\begin{aligned}\Delta_{\mu\nu}^{\alpha\beta} = &-\frac{1}{q_0}\int \mathrm{d}^3x \exp(-\mathrm{i}\boldsymbol{q}\,.\,\boldsymbol{x})\langle 0|[j_\mu{}^\alpha(x), j_\nu(0)]|0\rangle\delta(x_0) \\ &-\frac{1}{q_0{}^2}\int \mathrm{d}^3x \exp(-\mathrm{i}\boldsymbol{q}\,.\,\boldsymbol{x})\langle 0|[[j_\mu{}^\alpha(x), H]j_\nu{}^\beta(0)]|0\rangle\delta(x_0) + \mathrm{O}\left(\frac{1}{q^3}\right).\end{aligned} \tag{2.10}$$

Similarly taking the $q_0 \to \infty$ as limit for the right hand side of the expression (2.9), and identifying the $(1/q_0{}^2)$ term for Δ_{ij}, where $i = j = 1, 2, 3$, one obtains

$$\int \rho^{\alpha\beta}(m^2)\,\mathrm{d}m^2 = \int \mathrm{d}^3x\langle 0|[[j_i{}^\alpha(x), H]j_i{}^\beta(0)]|0\rangle. \tag{2.11}$$

There is no summation over the index i. We arrive at two model independent equations for the cross section $\sigma_{T=0}$ and $\sigma_{T=1}$

$$\frac{1}{16\pi^3\alpha^2}\int \mathrm{d}s\, s^2\sigma_{T=1}(s) = \int \mathrm{d}^3x\langle 0|[[j_i{}^3(x), H]j_i{}^3(0)]|0\rangle \tag{2.12}$$

$$\frac{3}{16\pi^3\alpha^2}\int \mathrm{d}s\, s^2\sigma_{T=0}(s) = \int \mathrm{d}^3x\langle 0|[[j_i{}^8(x), H]j_i{}^8(0)]|0\rangle. \tag{2.13}$$

To proceed further, it is necessary to make some models for the Hamiltonian H. Let us take the Hamiltonian first proposed by Gell-Mann (1964) and Gell-Mann *et al.* (1968)

$$\mathscr{H} = \mathscr{H}_0 - S_0 - cS_8 \tag{2.14}$$

where H_0 is the U(3) × U(3) invariant term, and S_0, S_8 are the singlet and the eighth component of the scalar density. The double commutators in equations (2.11) to (2.13) can then be evaluated in quark algebra to be

$$[[j_i^{\alpha}(x), S_{\beta}(y)]j_i^{\gamma}(0)] = -d_{\alpha\beta\delta}d_{\delta\gamma\epsilon}S_{\epsilon}\delta^3(x)\delta^3(x-y). \tag{2.15}$$

Then one has

$$\int \rho^{\alpha\alpha}(m^2)\,\mathrm{d}m^2 = d_{\alpha\alpha\gamma}\langle Z_{\gamma}\rangle + d_{0\alpha\gamma}d_{\gamma\alpha\delta}\langle S_{\delta}\rangle + cd_{8\alpha\gamma}d_{\gamma\alpha\delta}\langle S_{\delta}\rangle \tag{2.16}$$

where the first term is defined by

$$\int \mathrm{d}^3x\,\langle 0|[[j_i^{\alpha}(x), H_0]j_i^{\alpha}(0)]|0\rangle \equiv d_{\alpha\alpha\gamma}\langle Z_{\gamma}\rangle. \tag{2.17}$$

The quantity $\langle Z_{\gamma}\rangle$ will be discussed in more detail in the next section. Let us now parameterize the vacuum expectation values $\langle S_0\rangle$, $\langle S_8\rangle$ by

$$b = \frac{\langle S_8\rangle}{\langle S_0\rangle}. \tag{2.18}$$

The absolute magnitude of $\langle S_0\rangle$ can be related to physically measurable quantities by a sum rule (Mathur and Okubo 1970a, 1970b, Mathur *et al.* 1970, Kamal 1970, Lai and Lo, 1970, 1971, Auvil and Despande 1969 and Kamal 1969)

$$m_{\pi}^2F_{\pi}^2 = \tfrac{4}{3}\langle S_0\rangle\left(1+\frac{c}{\sqrt{2}}\right)\left(1+\frac{b}{\sqrt{2}}\right) \tag{2.19}$$

where F_{π} is the pion decay constant approximately equal to 0·130 GeV. Evaluating equation (2.16) with the aid of equations (2.18) and (2.19), one gets

$$\int (\rho^{33}(m^2)-\rho^{88}(m^2))\,\mathrm{d}m^2 = \frac{2}{\sqrt{3}}\langle Z_8\rangle + \bar{\alpha}F_{\pi}^2m_{\pi}^2 \tag{2.20}$$

where

$$\bar{\alpha} = \frac{b+c-bc/\sqrt{2}}{\sqrt{2}\{1+(c/\sqrt{2})\}\{1+(b/\sqrt{2})\}}. \tag{2.21}$$

In the simple case where $\langle Z_8\rangle = 0$ we have a favourable sum rule†

$$\frac{1}{16\pi^3\alpha^2}\int_0^{\infty} \mathrm{d}s\,s^2(\sigma_{T=1}(s)-3\sigma_{T=0}(s)) = \bar{\alpha}F_{\pi}^2m_{\pi}^2. \tag{2.22}$$

The difference between the cross sections $\sigma_{T=1}-3\sigma_{T=0}$ gives a direct measure of the U(3)⊗U(3) symmetry breaking parameters b, c.

3. Estimate of $\langle Z_8\rangle$

The value of $\langle Z_8\rangle$ is quite crucial to the evaluation of our equation (1.3). It is in general rather model dependent, and we shall give the estimates of $\langle Z_8\rangle$ under three different assumptions: (i) the asymptotic SU(3) symmetry (Das *et al.* 1967a, 1967b); (ii) the saturation of the spectrum functions with mesons; (iii) the free quark model.

† Numerically $16\alpha^2\pi^3F_{\pi}^2m_{\pi}^2 \simeq 3{\cdot}3$ nb GeV^6 which is not insignificant for e^+e^- annihilation.

3.1. *Assumption* (i): *asymptotic* SU(3) *symmetry*

The asymptotic SU(3) symmetry means (Das *et al.* 1967a, 1967b)

$$\lim_{q_0\to\infty} \Delta^{33}_{\mu\nu}(q^2) = \lim_{q_0\to\infty} \Delta^{88}_{\mu\nu}(q^2). \tag{3.1}$$

Identifying the $(1/q_0)^2$ term, one obtains from equation (2.10)

$$\int d^3x \exp(-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x}) \langle 0|[[J_\mu{}^3(x), H], J_\nu{}^3(0)]|0\rangle \delta(x_0)$$

$$= \int d^3x \exp(-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x}) \langle 0|[[J_\mu{}^8(x), H], J_\nu{}^8(0)]|0\rangle \delta(x_0)$$

at $\boldsymbol{q} = 0$, one gets

$$\int \rho^{33}(m^2)\, dm^2 = \int \rho^{88}(m^2)\, dm^2$$

or

$$\int_0^\infty ds\, s^2(\sigma_{T=1}(s) - 3\sigma_{T=0}(s)) = 0. \tag{3.2}$$

The value of $\langle Z_8\rangle$ is

$$\langle Z_8\rangle = -\frac{\sqrt{3}}{2}\bar{\alpha} F_\pi{}^2 m_\pi{}^2. \tag{3.3}$$

3.2. *Assumption* (ii): *saturation of the spectrum functions with mesons*

An additional sum is necessary to help us to estimate $\langle Z_8\rangle$. It can be derived by following essentially the steps (2.9) to (2.10), (2.16) for the axial vector current (Auvil and Despande 1969, Kamal 1969)

$$\int \rho^{\alpha\alpha}(m^2)\, dm^2 = d_{\alpha\alpha\gamma}\langle Z_\gamma\rangle + \int \rho_{A0} m^2\, dm^2 \tag{3.4}$$

where ρ_{A0} is the pseudoscalar spectrum function. Using the notation of Kroll *et al.* (1967) we have

$$\rho^{33}(m^2) = \left(\frac{m_\rho{}^2}{f_\rho}\right)^2 \delta(m^2 - m_\rho{}^2)$$

$$\rho^{88}(m^2) = \frac{3}{4f_Y{}^2}\{\cos^2\theta_Y m_\phi{}^2\delta(m^2 - m_\phi{}^2) + \sin^2\theta_Y m_\omega{}^2\delta(m^2 - m_\omega{}^2)\} \text{ etc} \tag{3.5}$$

where we saturate the spectrum functions with mesons. We get for $\alpha = 3$, and 4

$$\left(\frac{m_\rho{}^2}{f_\rho{}^2}\right)^2 = d_{33\gamma}\langle Z_\gamma\rangle + m_\pi{}^2 F_\pi{}^2 \tag{3.6}$$

$$\left(\frac{m_{K*}{}^2}{f_{K*}}\right)^2 = d_{44\gamma}\langle Z_\gamma\rangle + m_K{}^2 F_K{}^2. \tag{3.7}$$

With the equality

$$\frac{m_\rho{}^2}{f_\rho{}^2} = \frac{m_{K*}{}^2}{f_{K*}{}^2} \tag{3.8}$$

where, following Sakurai (1969), the K meson's contributions are neglected, one obtains

$$\frac{2}{\sqrt{3}}\langle Z_8\rangle = \bar{\alpha}_8 F_\pi{}^2 m_\pi{}^2 \tag{3.9}$$

with

$$\bar{\alpha}_8 = \frac{4}{3m_\pi{}^2}\left\{2(m_\rho{}^2 - m_{\mathrm{K}^*}{}^2) - \left(m_\pi{}^2 - m_{\mathrm{K}^*}{}^2\frac{F_{\mathrm{K}}{}^2}{F_\pi{}^2}\right)\right\}$$
$$\simeq -0{\cdot}9. \tag{3.10}$$

Hence $\bar{\alpha}_8$ is a small number compared with $\bar{\alpha}$, which is evaluated in the next section.

3.3. *Assumption* (iii): *free quark model*

In the simple quark model without any interaction $\mathscr{H}_0$ is simply the kinetic energy term

$$\mathscr{H}_0 = \bar{q}(\nabla\gamma)q. \tag{3.11}$$

Then $\langle Z_\gamma\rangle$ can actually be evaluated to yield

$$\langle Z_\gamma\rangle = \int\frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{Tr}\{(\gamma_i k_i - \boldsymbol{k}\,.\,\gamma)S(k)\lambda_\gamma\}$$

where

$$S_{ab}(k^2) = \delta_{ab}\int \mathrm{d}m^2\left(-\frac{\mathrm{i}\rho_1{}^a(m^2)\gamma.k + \rho_2{}^a(m^2)}{k^2+m^2}\right) \tag{3.12}$$

with $a, b = 1, 2, 3$ as the quark indices and $\rho_{1,2}{}^a(m^2)$ are quark spectrum functions. There are two possibilities for the quark spectrum function: (a) $\rho_1{}^a \neq \rho_1{}^b$ for $a \neq b$ $\langle Z_8\rangle$ will not vanish but actually diverge. If $\rho_1{}^a$ are those of bare quarks:

$$\rho_1{}^a \simeq \delta(m^2 - m_a{}^2) \tag{3.13}$$

and $m_1 = m_2 \neq m_3$ then

$$\langle Z_\gamma\rangle \simeq \int\frac{\mathrm{d}^4k}{k^2} \tag{3.14}$$

diverges linearly with k^2. From equation (1.3) one gets

$$\sigma_{T=1}(s) - \sigma_{T=0}(s) \simeq \frac{1}{s^2}. \tag{3.15}$$

(b) $\rho_1{}^1 = \rho_1{}^2 = \rho_1{}^3$. Then we have $\langle Z_8\rangle = 0$ and the sum rule (equation (2.22)) holds.

4. Numerical estimates of $\bar{\alpha}$

Various authors (Gell-Mann *et al.* 1968, Mathur and Okubo 1970a, 1970b, Mathur *et al.* 1970, Kamal 1970, Lai and Lo 1970, 1971) have assigned quite different values to b and c. We shall try to calculate $\bar{\alpha}$ from some of them.

(i) The Gell-Mann–Oakes–Renner estimate: they have set $b \simeq 0$, and $c = -1{\cdot}25$. Hence we get

$$\bar{\alpha} \simeq \frac{1}{\sqrt{2}}\frac{c}{1+c/\sqrt{2}} \simeq -7{\cdot}6. \tag{4.1}$$

(ii) Model A (Lai and Lo 1971): in a previous paper we have solved consistently sum rules involving pseudoscalar mesons. The pseudoscalar mesons are taken to be eigenstates of the quark pseudoscalar density. The result is

$$\begin{aligned} b = c &\simeq -0{\cdot}97 \\ \bar{\alpha} &= \frac{c(2-c/\sqrt{2})}{\sqrt{2}(1+c/\sqrt{2})^2} \\ &\simeq -18{\cdot}6. \end{aligned} \tag{4.2}$$

(iii) Model B (Lai and Lo 1970): this is a variant of model A. All the assumptions are the same as in model A except that the pseudoscalar mesons are taken to be the eigenstates of the axial vector currents. Then we have

$$b+c = \frac{bc}{2} \tag{4.3}$$

and hence

$$\bar{\alpha} = 0. \tag{4.4}$$

The sum rule becomes

$$\frac{1}{16\pi^3\alpha^2}\int_0^\infty ds\, s^2(\sigma_{T=1}(s)-3\sigma_{T=0}(s)) = \bar{\alpha}_8 F_\pi{}^2 m_\pi{}^2. \tag{4.5}$$

It should be reminded that the values b, c in this model do not fit into the groups favoured by Okubo and Mathur (1970a).

(iv) Saturation of spectrum function with vector mesons: if the spectrum function sum rules can be saturated with vector mesons, one has a different way of evaluating $\bar{\alpha}$ quite independently of the values b, c. Substituting equation (3.5) in equation (2.20), and together with equation (3.9), one gets

$$\bar{\alpha} = \frac{m_\rho{}^2}{m_\pi{}^2 F_\pi{}^2 f_\rho{}^2}\left(m_\rho{}^2 - \frac{\cos^2\theta_Y m_\phi{}^4 + \sin^2\theta_Y m_\omega{}^4}{\cos^2\theta_Y m_\phi{}^2 + \sin^2\theta_Y m_\omega{}^2}\right) - \bar{\alpha}_8.$$

It can be estimated by the experimentally determined quantities (Perez-y-Jorba 1969)

$$\theta_Y = 41{\cdot}5^\circ \qquad \frac{f_\rho{}^2}{4\pi} = 2{\cdot}0 \qquad F_\pi = 0{\cdot}130 \text{ GeV}$$
$$m\rho^2 = 0{\cdot}599 \text{ GeV}^2$$

to be

$$\bar{\alpha} \cong -21.$$

It is remarkable that this value comes close to the value obtained in model A. Hence the vector meson dominance model strongly supports the symmetry breaking values found in model A.

To summarize the results in §§ 3 and 4, we present various estimates in table 1. The last column contains the measurable quantities. The values from different models are sufficiently divergent that one may be able to differentiate between them experimentally.

Table 1

Model	$\bar{\alpha}_8$	$\bar{\alpha}$	$\bar{\alpha}_8+\bar{\alpha} = (16\pi^3\alpha^2F_\pi^2m_\pi^2)^{-1} \times \int_0^\infty ds\, s^2(\sigma_{T=1}(s)-\sigma_{T=0}(s))$
(i) Asymptotic SU(3) symmetry	$-\bar{\alpha}$	—	0
(ii) Saturation of vector meson dominance	$-0{\cdot}93$	-21	$-21{\cdot}8$
(iii) Free quark model			
(*a*)	0	—	—
(*b*)	∞	—	∞
(i) Gell-Mann–Oakes–Renner	—	$-7{\cdot}6$	$-7{\cdot}6$†
(ii) Model A	—	$-18{\cdot}6$	-18.6†
(iii) Model B	—	0	0†

† It is evaluated together with the value $\bar{\alpha}_8 = 0$.

5. Concluding remarks

(i) We hope we have demonstrated that the measurement of the difference of $\sigma_{T=1}$ and $\sigma_{T=0}$ offers a good method of determining the U(3) × U(3) symmetry breaking effects. We have only considered the symmetry breaking type of $(3, 3^*)\otimes(3^*, 3)$. If the type $(1, 8)\otimes(8, 1)$ also exists, it could easily be incorporated in equations (2.12) and (2.13), by adding an additional term to the Hamiltonian in equation (1.2).

(ii) We do not know how rapidly the integral of the sum rule (1.3) converges. However, if the integral actually diverges, it should be noticed quite early, and the theoretical significance is rather interesting as discussed at the end of § 3.

(iii) The saturation of vector mesons of the spectrum function gives support to the values $b = c \simeq -1$ for the symmetry breaking parameters. These are the preferred values of our calculations from quite different considerations (Lai and Lo 1970, 1971).

(iv) The separation of the e^+e^- annihilation cross section into $\sigma_{I=1}$ and $\sigma_{I=0}$ is not so difficult for some cases, for example, the $\sigma_{I=1}$ contains $\pi\pi$, $\pi\phi$, $\pi\omega$, $\eta\rho$, Xρ etc, and $\sigma_{I=0}$ contains $\rho\pi$, $\omega\eta$, $\phi\eta$ etc, as their final states respectively.

In most cases the separation is difficult and one may have to reconcile oneself to some more delicate procedures to separate $\sigma_{I=1}$ and $\sigma_{I=0}$ such as suggested by Pais and Treiman (1970).

(iv) Finally, we wish to point out that the sum rule (1.3) is obtained through the use of the full chiral U(6)⊗U(6) algebra of current densities.

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